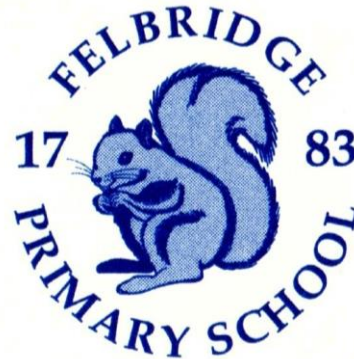


Felbridge Primary School Calculation Policy

Review date: Annually (autumn term)

Reviewed: December 2019



Felbridge Primary School

Crawley Down Road

East Grinstead

Surrey

RH10 4AL

“Tell me and I forget. Teach me and I remember. Involve me and I learn.” – Benjamin Franklin

This policy is a framework of expectations for how children should be taught to develop their understanding of the 4 operations at Felbridge. It has been developed to support a teaching for mastery approach. Solid mathematical understanding in every child at Felbridge is underpinned by each child’s journey through the concrete → pictorial → abstract (written methods). This policy is a guide through the appropriate progression of written calculation and if at any time a child is struggling with the abstract a child should revert to the pictorial or concrete to aid their solving of problems, where appropriate. It is vital that children are using a strategy that is appropriate to their **stage of learning**. As a result this may result in them using a strategy that is found in a different age group.



Five Principles of Number

Principle	Success criteria	Context
Stable order principle	Can say some number names when asked to count.	Counting objects as they are put out on a table for art, role play, games.... Counting children in a group. Counting around a group up to a target number.
	Can join in with saying number names in order.	
	Can say number names in order to 10 starting with 0.	
	Can say number names in order to 20 starting with 0.	
One to one principle	Can point to objects as a number name is being said.	Moving counting objects from a pot into a tub as they are counted. Holding objects in hand and placing them down on the table one by one saying the number each time. Counting beads along a bead string.
	Can move objects as the number names are being said one at a time.	
	Can point to each object (or move it) only once as it is being counted.	
Cardinal principle	Can respond to “how many?” by saying number names in order and knowing last number said is how many.	Using pointing or moving strategy count sets of counters, pencils, paperclip, leaves, bean bags...
	Can repeat how many are in the set without having to recount it.	
Order irrelevance principle	Can say how many are in a set despite having the set rearranged between requests.	Practise making and moving sets of objects without adding or taking any away. Make patterns and pictures using counted sets. Make sets using objects of mixed varying sizes.
Abstraction principle	Can count a series of claps, coin drops (to 10/20).	Practise saying number names in order to a signal such as a clap, wave, nod... Count actions as well as objects, count words on a page, words spoken, foot tapped... Play “my turn your turn” for showing a target number.
	Can count a series of own actions, e.g. jumps, clap?	

Place value should only be taught once the five principles of number are secure.

Vocabulary

Children should be introduced to the correct mathematical language at the earliest opportunity. The following language can be used within calculations.

Addend- a number which is added to another

Sum/Total- the total amount resulting from the addition of two or more numbers, amounts, or items.

Minuend- a quantity or number from which another is to be subtracted.

Subtrahend- a quantity or number to be subtracted from another.

Difference- the result of subtracting one number from another.

Multiplicand- a quantity which is to be multiplied by another

Multiplier- a quantity by which a given number is to be multiplied.

Product- the result of multiplying.

Dividend- a number to be divided by another number.

Divisor- a number by which another number is to be divided.

Quotient- a result obtained by dividing one quantity by another.

$$\text{Addend} + \text{addend} = \text{sum or total}$$

$$\text{Minuend} - \text{subtrahend} = \text{difference}$$

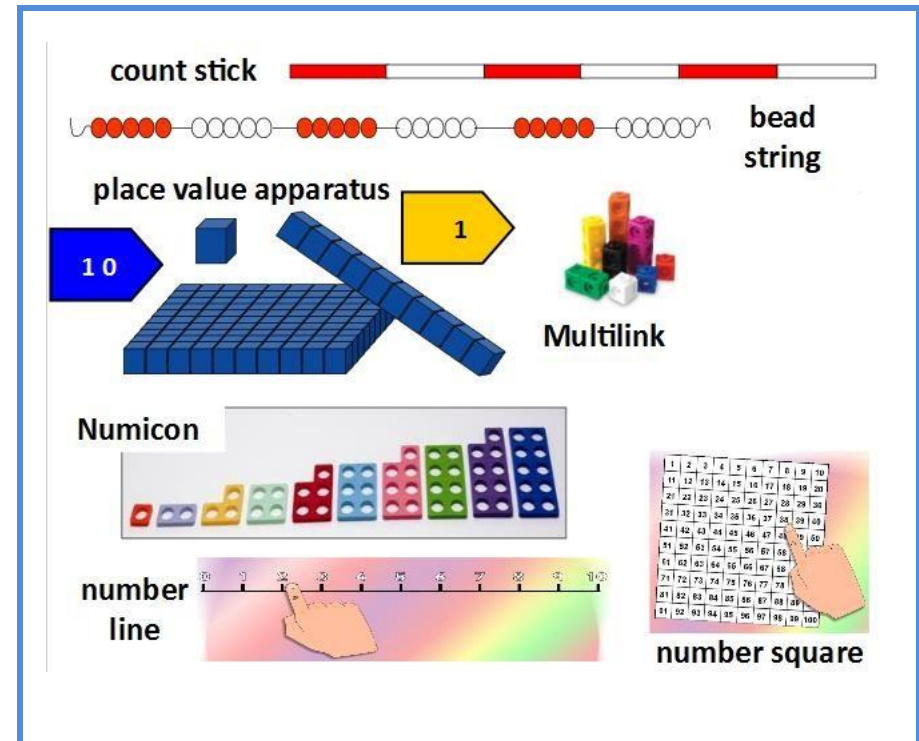
$$\text{Multiplicand} \times \text{multiplier} = \text{product}$$

$$\text{Dividend} \div \text{divisor} = \text{quotient}$$

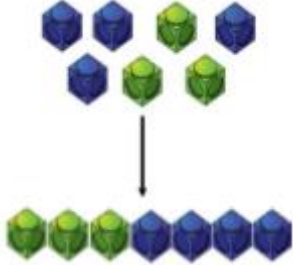

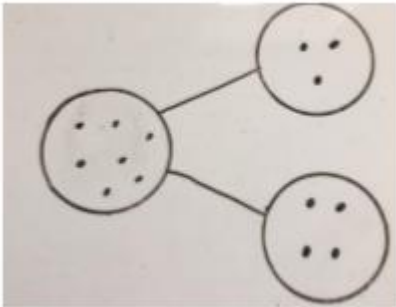
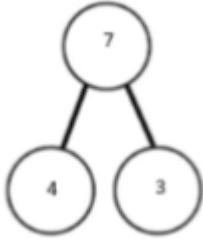
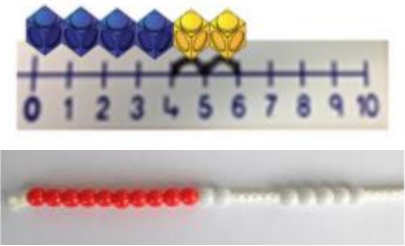
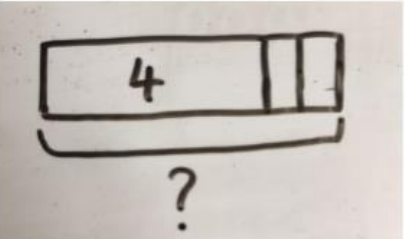

Subject specific language can be found at the end of each calculation section.

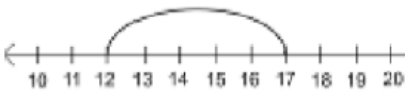
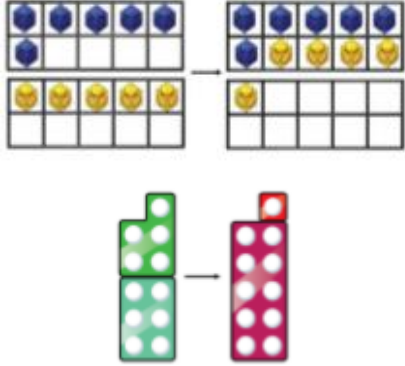
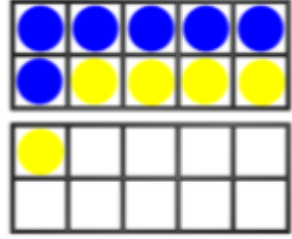
Resources

A range of resources may be used however the following should be available to all children.



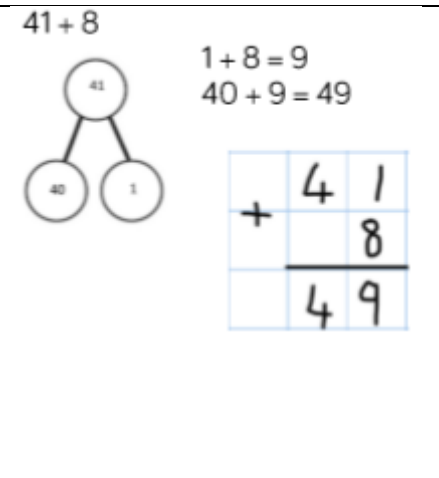
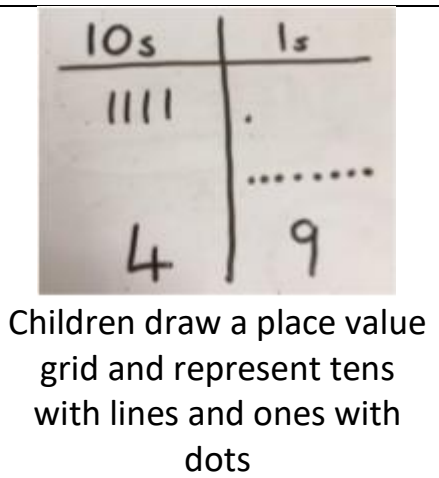
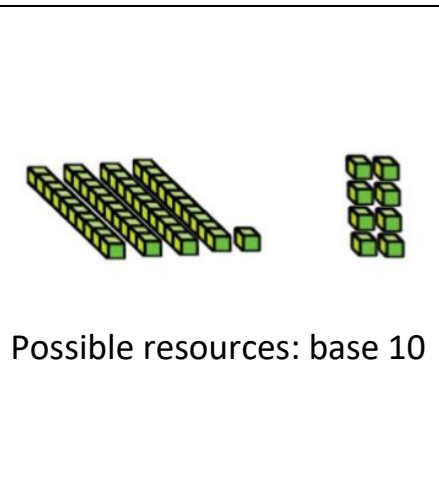
Written Methods for Addition

YEAR GROUP & RELEVANT OBJECTIVES	STRATEGY	CONCRETE	PICTORIAL	ABSTRACT / WRITTEN
<p>Y1: Add one-digit numbers to 20 including 0</p> <p>Y1: Add two-digit numbers to 20</p> <p>Y2: Add numbers using concrete objects and pictorial representations, including adding three one-digit numbers</p>	<p>Aggregation – combining two parts to make a whole</p>	<p>$3+7=$</p>  <p>Possible resources: cubes, numicon, teddies, etc.</p>	  <p>Part-whole model where the numbers are represented by dots</p>	<p>(N/A for Class R)</p> <p>$4 + 3 = 7$</p> 
<p>Y1: Add one-digit numbers to 20 including 0</p> <p>Y1: Add two-digit numbers to 20</p> <p>Y2: Add numbers using concrete objects and pictorial representations, including adding</p>	<p>Augmentation – increasing a quantity by an amount (starting with the largest number and counting on)</p>		 <p>Bar model which encourages children to</p>	<p>$4 + 2 = 6$</p>  <p>The abstract number line.</p>

<p>three one-digit numbers</p>		<p>Possible resources: bead string, number lines with cubes or numicon</p>	<p>count on, rather than count all</p>  <p>Counting on using a number line, beginning at the largest number and counting on in ones or in one jump</p>	
<p>Y1: Add one-digit numbers to 20 including 0 Y1: Add two-digit numbers to 20 Y2: Add numbers using concrete objects and pictorial representations, including adding three one-digit numbers</p>	<p>Regrouping – i.e. to make 10</p>	 <p>Possible resources: ten frames and cubes, numicon</p>	 <p>Children draw their own ten frames and dots</p>	<p>Children develop an understanding of equality and look for links between numbers.</p> $6 + [] = 11$ $6 + 5 = 5 + []$ $6 + 5 = [] + 4$ $11 = [] + 6$

Y2: Add numbers using concrete objects and pictorial representations, including a two-digit number and ones

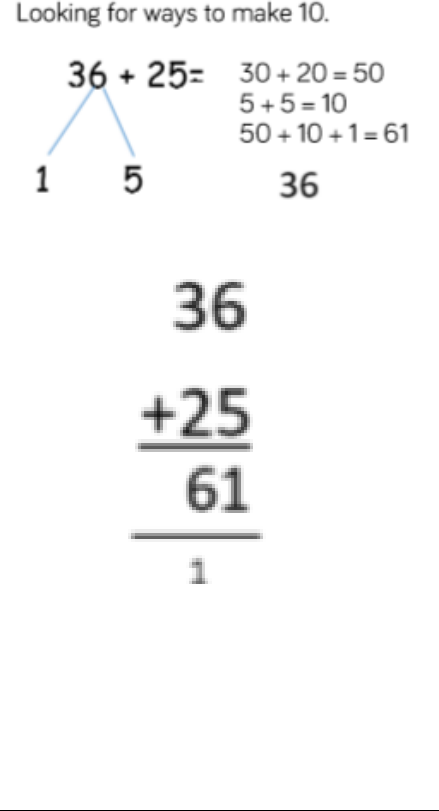
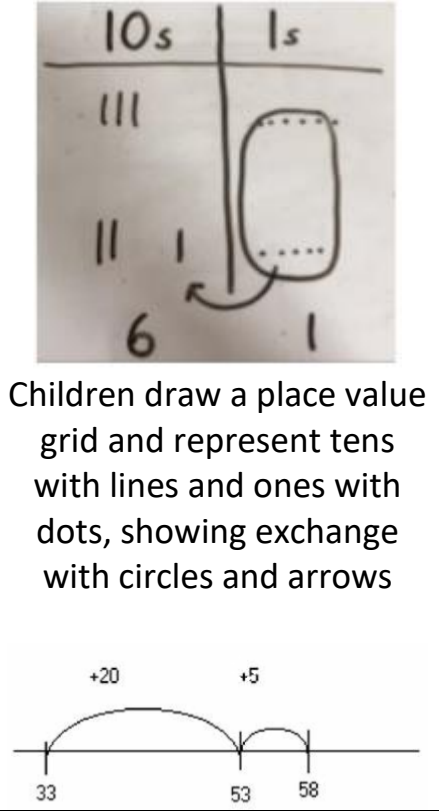
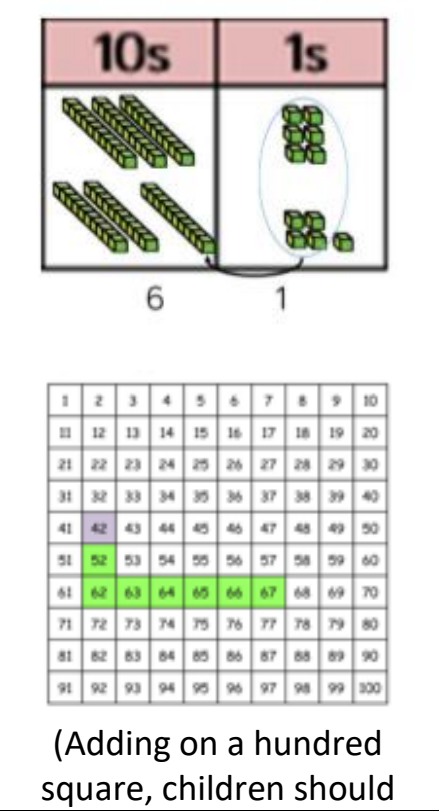
TO + O – developing understanding of place value and partitioning
 Step 1: without exchange
 Step 2: with exchange

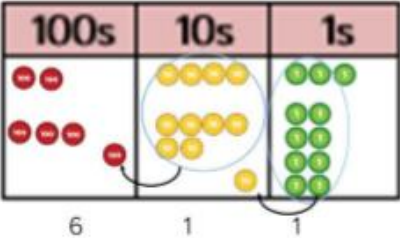
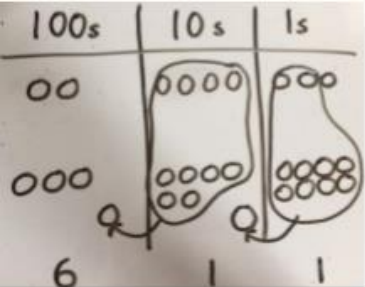


Y2: Add numbers using concrete objects and pictorial representations, including a two-digit number and tens

 Y2: Add numbers using concrete objects and pictorial representations, including two two-digit numbers

TO + TO – continue to develop understanding of place value and partitioning
 Step 1: without exchange
 Step 2: with exchange

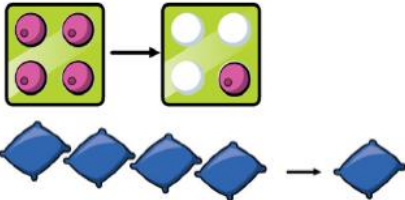
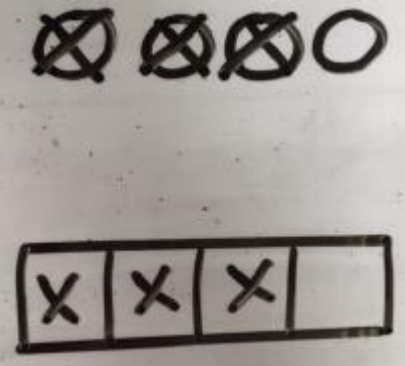
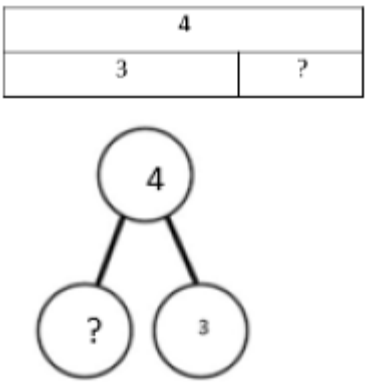

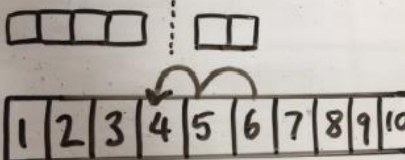


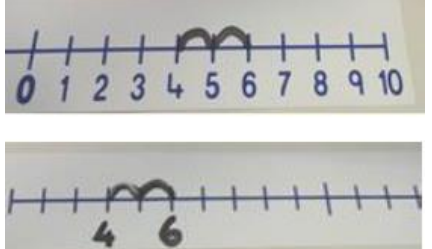
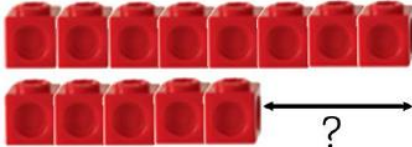

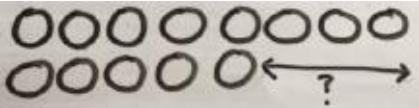
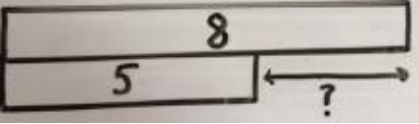
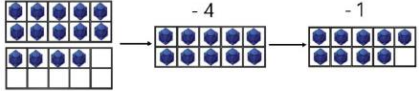
		<p>move down to add tens and along to add ones)</p> <p>Possible resources: place value grid, base 10, hundred square</p>	<p>Using an empty number line, counting in jumps of tens and ones</p>	
<p>Y3: Add numbers with up to three digits, using formal written methods of columnar addition</p> <p>Y4: Add numbers with up to 4 digits using the formal written methods of columnar addition where appropriate</p> <p>Y5: Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)</p>	<p>HTO + TO or HTO + HTO (with multiple exchanges)</p> <p>Any number of digits + any number of digits, (with multiple exchanges)</p> <p>Adding decimals</p> <p>Adding more than 2 numbers</p>	 <p>Possible resources: place value grid, base 10, place value counters</p>	 <p>Children draw a place value grid and represent tens with lines and ones with dots, showing exchange with circles and arrows</p>	$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 11 \end{array}$

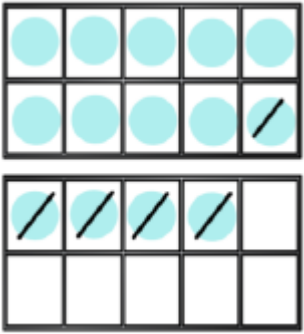
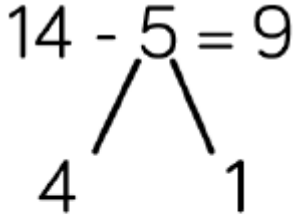
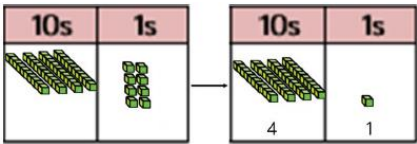
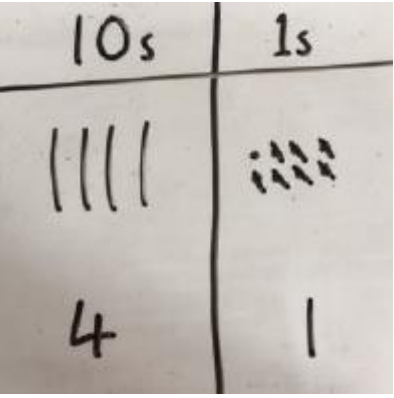
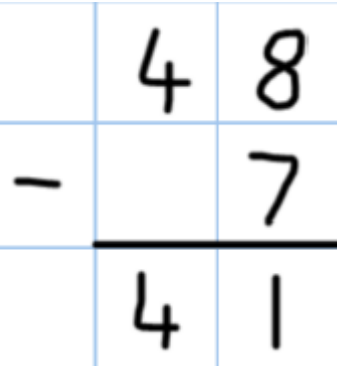
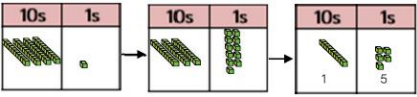
Key vocabulary for addition:

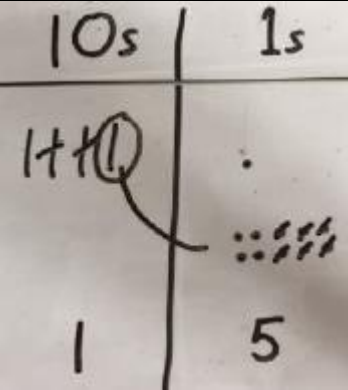
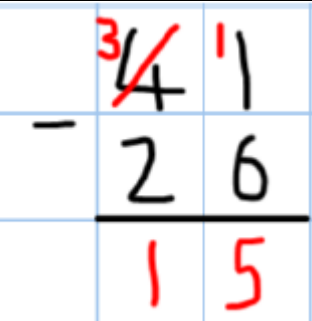
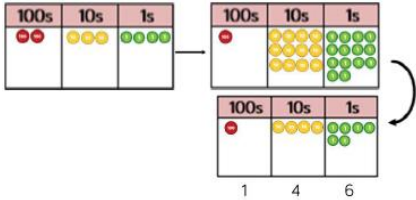
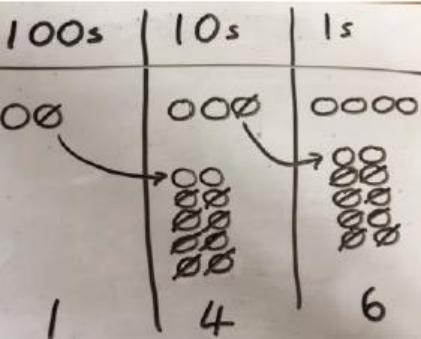
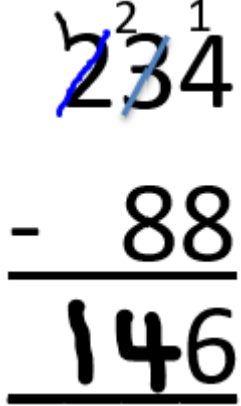
sum, total, parts and wholes, plus, add, altogether, more, 'is equal to', 'is the same as

Written Methods for Subtraction

YEAR GROUP & RELEVANT OBJECTIVES	STRATEGY	CONCRETE	PICTORIAL	ABSTRACT / WRITTEN
<p>Y1: Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.</p> <p>Y2: Solve problems with addition and subtraction : using concrete objects and pictorial representations, including those involving numbers, quantities and measures, applying their increasing knowledge of mental and written methods.</p>	<p>Partitioning Taking away and removing objects from original set</p>	<p>$4 - 3 = 1$</p>  <p>Possible resources: numicon, bean bags, cubes, tens frame</p>	<p>Children draw resources and cross out</p> 	<p>$4 - 3 =$ $_ = 4 - 3$</p> 
<p>Y1: Subtract one digit and two- digit numbers</p> <p>Y2: Add and subtract numbers using</p>	<p>Reduction Start at and count back</p>	<p>$6 - 2 = 4$</p> 	<p>Draw what they see</p> 	<p>Children represent on a numberline or track, progressing to empty line</p>

<p>concrete objects, pictorial representations and mentally</p>		<p>Possible resources: cubes or number tracks</p>		
<p>Y1: Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.</p> <p>Y2: Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</p>	<p>Comparison Finding the difference between two numbers</p>	<p>$8 - 5 = 3$</p>   <p>Possible resources: cubes, base 10, numicon</p>	<p>Children draw the cubes or objects, bar model can also be used to show what they need to calculate</p>  	<p>Find the difference between 8 and 5.</p> <p>$8 - 5$, the difference is <input type="text"/></p> <p>Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.</p>
<p>Y1: Represent and use number bonds and related subtraction facts within 20</p> <p>Y2: Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</p>	<p>Making 10 (bridging 10)</p>	<p>$14 - 5 = 9$</p>  <p>Possible resources: number tracks, number lines, 10s frame, numicon</p>	<p>Children represent 10s frame pictorially. Children should be encouraged to explain what they have done</p>	<p>Children demonstrate partitioning of subtrahend</p>

				$14 - 5 = 9$  $14 - 4 = 10$ $10 - 1 = 9$
<p>Y1: Subtract one digit and two-digit numbers</p> <p>Y2: Add and subtract numbers using concrete objects, pictorial representations and mentally</p> <p>Y3: add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</p>	<p>Column method T U</p>	<p>$48 - 7 = 41$</p>  <p>Possible resources: base 10, PV counters</p>	<p>Children represent base 10 with I and x</p> 	<p>Children use column method and apply number facts to 10</p> 
<p>Y3: add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</p>	<p>Column method (with exchanging, decomposition) T U</p>	<p>$41 - 26 = 15$</p>  <p>Using base 10 with exchange</p>	<p>Children draw with I and x, remembering to show exchange</p>	<p>Formal column method to be used (CHILDREN MUST SHOW UNDERSTANDING OF DECOMPOSITION AND THAT THEY still have 41 as $41 = 30 + 11$)</p>

<p>Y4: add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate</p> <p>Y5: add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)</p>				
<p>Y3: add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction</p> <p>Y4: add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate</p> <p>Y5: add and subtract whole numbers with more than 4 digits, including using formal written</p>	<p>Column method (with PV counters) HTU</p>	<p>$234 - 88 = 146$</p>  <p>Using place value counters</p>	<p>Children to draw place value counters, showing what has been exchanged</p> 	<p>Formal column method to be used (CHILDREN MUST SHOW UNDERSTANDING OF DECOMPOSITION)</p> 

methods (columnar addition and subtraction)				
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Key vocabulary for subtraction:

take away, less than, the difference, subtract, minus, fewer, decrease, subtrahend

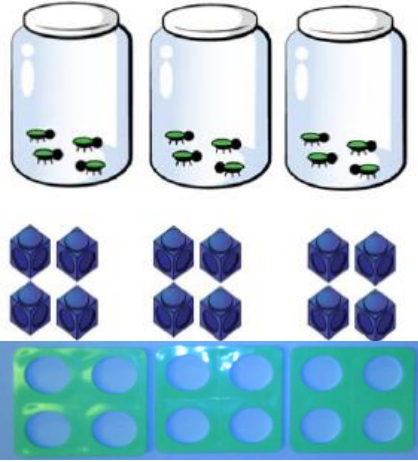
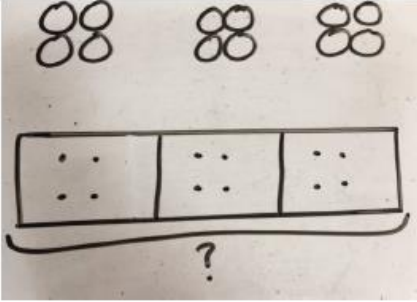
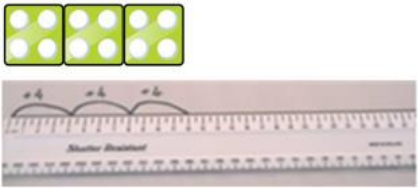
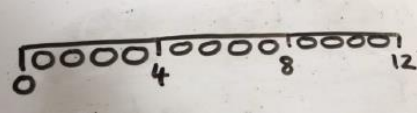
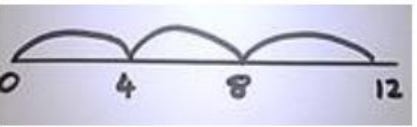
Mental Expectations for Addition and Subtraction

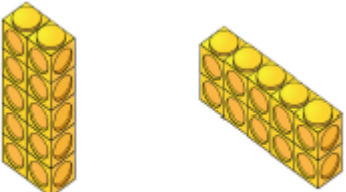
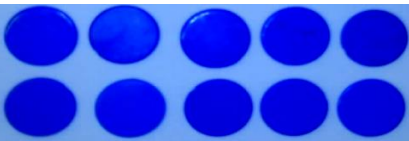
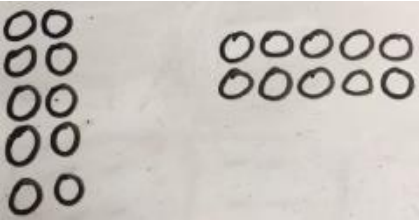
<u>By the end of the year</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
<u>-children are able to derive and recall</u>	<ul style="list-style-type: none"> number pairs with a total of 10, e.g. $3 + 7$, or what to add to a single-digit number to make 10, e.g. $3 + \square = 10$ addition facts for totals to at least 5, e.g. $2 + 3$, $4 + 3$ addition doubles for all numbers to at least 10, e.g. $8 + 8$ 	<ul style="list-style-type: none"> addition and subtraction facts for all numbers up to at least 10, e.g. $3 + 4$, $8 - 5$ number pairs with totals to 20 all pairs of multiples of 10 with totals up to 100, e.g. $30 + 70$, or $60 + \square = 100$ what must be added to any two-digit number to make the next multiple of 10, e.g. $52 + \square = 60$ addition doubles for all numbers to 20, e.g. $17 + 17$ and multiples of 10 to 50, e.g. $40 + 40$ 	<ul style="list-style-type: none"> addition and subtraction facts for all numbers to 20, e.g. $9 + 8$, $17 - 9$, drawing on knowledge of inverse operations sums and differences of multiples of 10, e.g. $50 + 80$, $120 - 90$ pairs of two-digit numbers with a total of 100, e.g. $32 + 68$, or $32 + \square = 100$ addition doubles for multiples of 10 to 100, e.g. $90 + 90$ 	<ul style="list-style-type: none"> sums and differences of pairs of multiples of 10, 100 or 1000 addition doubles of numbers 1 to 100, e.g. $38 + 38$, and the corresponding halves what must be added to any three-digit number to make the next multiple of 100, e.g. $521 + \square = 600$ pairs of fractions that total 1 	<ul style="list-style-type: none"> sums and differences of decimals, e.g. $6.5 + 2.7$, $7.8 - 1.3$ doubles and halves of decimals, e.g. half of 5.6, double 3.4 what must be added to any four-digit number to make the next multiple of 1000, e.g. $4087 + \square = 5000$ what must be added to a decimal with units and tenths to make the next whole number, e.g. $7.2 + \square = 8$ 	<ul style="list-style-type: none"> addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place, e.g. $650 + \square = 930$, $\square - 1.4 = 2.5$ what must be added to a decimal with units, tenths and hundredths to make the next whole number, e.g. $7.26 + \square = 8$
<u>-working mentally (with jottings where necessary)</u>	<ul style="list-style-type: none"> add or subtract a pair of single-digit numbers, e.g. $4 + 5$, $8 - 3$ add or subtract a single-digit number to or from a teens number, e.g. $13 + 5$, $17 - 3$ add or subtract a single-digit to or from 10, and add a multiple of 10 to a single-digit number, e.g. $10 + 7$, $7 + 30$ 	<ul style="list-style-type: none"> add or subtract a pair of single-digit numbers, including crossing 10, e.g. $5 + 8$, $12 - 7$ add any single-digit number to or from a multiple of 10, e.g. $60 + 5$ subtract any single-digit number from a multiple of 10, e.g. $80 - 7$ add or subtract a single-digit number 	<ul style="list-style-type: none"> add and subtract groups of small numbers, e.g. $5 - 3 + 2$ add or subtract a two-digit number to or from a multiple of 10, e.g. $50 + 38$, $90 - 27$ add and subtract two-digit numbers, e.g. $34 + 65$, $68 - 35$ add near doubles, e.g. $18 + 16$, $60 + 70$ 	<ul style="list-style-type: none"> add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. $47 + 58$, $91 - 35$ add or subtract a near multiple of 10, e.g. $56 + 29$, $86 - 38$ add near doubles of two-digit numbers, e.g. $38 + 37$ add or subtract two-digit or three-digit multiples of 10, e.g. 	<ul style="list-style-type: none"> add or subtract a pair of two-digit numbers or three-digit multiples of 10, e.g. $38 + 86$, $620 - 380$, $350 + 360$ add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. $235 + 198$ find the difference between near multiples of 100, e.g. 	<ul style="list-style-type: none"> count on or back in hundreds, tens, ones and tenths partition: add hundreds, tens or ones separately, then recombine subtract by counting up from the smaller to the larger number add or subtract a multiple of 10 or 100 and adjust partition: double and adjust

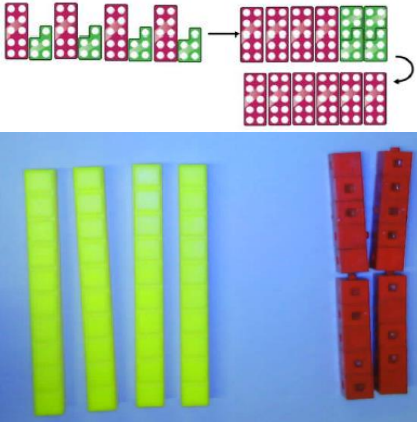
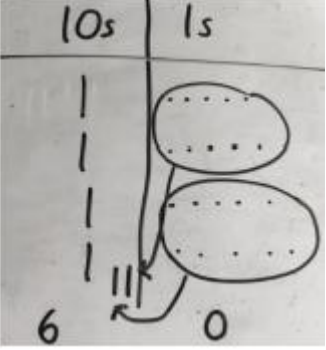
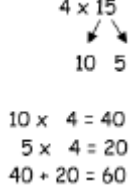
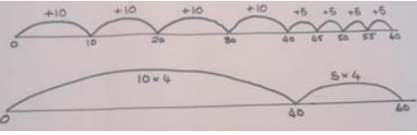
	<ul style="list-style-type: none"> • add near doubles, e.g. $6 + 7$ 	<p>to or from a two-digit number, including crossing the tens boundary, e.g. $23 + 5$, $57 - 3$, then $28 + 5$, $52 - 7$</p> <ul style="list-style-type: none"> • add or subtract a multiple of 10 to or from any two-digit number, e.g. $27 + 60$, $72 - 50$ add 9, 19, 29, ... or 11, 21, 31, ... • add near doubles, e.g. $13 + 14$, $39 + 40$ 		<p>$120 - 40$, $140 + 150$, $370 - 180$</p>	<p>$607 - 588$, or of 1000, e.g. $6070 - 4087$</p> <ul style="list-style-type: none"> • add or subtract any pairs of decimal fractions each with units and tenths, e.g. $5.7 + 2.5$, $6.3 - 4.8$ 	<ul style="list-style-type: none"> • use knowledge of place value and related calculations, e.g. $6.3 - 4.8$ using $63 - 48$ • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)
<p><u>-understand when to be able to apply</u></p>	<ul style="list-style-type: none"> • reorder numbers when adding, e.g. put the larger number first • count on or back in ones, twos or tens • partition small numbers, e.g. $8 + 3 = 8 + 2 + 1$ • partition and combine tens and ones • partition: double and adjust, e.g. $5 + 6 = 5 + 5 + 1$ 	<ul style="list-style-type: none"> • reorder numbers when adding • partition: bridge through 10 and multiples of 10 when adding and subtracting • partition and combine multiples of tens and ones • use knowledge of pairs making 10 • partition: count on in tens and ones to find the total • partition: count on or back in tens and ones to find the difference • partition: add a multiple of 10 and adjust by 1 • partition: double and adjust 	<ul style="list-style-type: none"> • reorder numbers when adding • identify pairs totalling 10 or multiples of 10 • partition: add tens and ones separately, then recombine • partition: count on in tens and ones to find the total • partition: count on or back in tens and ones to find the difference • partition: add or subtract 10 or 20 and adjust • partition: double and adjust • partition: count on or back in minutes and hours, bridging through 60 (analogue times) 	<ul style="list-style-type: none"> • count on or back in hundreds, tens and ones • partition: add tens and ones separately, then recombine • partition: subtract tens and then ones, e.g. subtracting 27 by subtracting 20 then 7 • subtract by counting up from the smaller to the larger number • partition: add or subtract a multiple of 10 and adjust, e.g. $56 + 29 = 56 + 30 - 1$, or $86 - 38 = 86 - 40 + 2$ • partition: double and adjust • use knowledge of place value and related calculations, e.g. work out $140 +$ 	<ul style="list-style-type: none"> • add or subtract pairs of decimals with units, tenths or hundredths, e.g. $0.7 + 3.38$ • find doubles of decimals each with units and tenths, e.g. $1.6 + 1.6$ • add near doubles of decimals, e.g. $2.5 + 2.6$ • add or subtract a decimal with units and tenths, that is nearly a whole number, e.g. $4.3 + 2.9$, $6.5 - 3.8$ 	<ul style="list-style-type: none"> • count on or back in hundreds, tens, ones, tenths and hundredths • use knowledge of place value and related calculations, e.g. $680 + 430$, $6.8 + 4.3$, $0.68 + 0.43$ can all be worked out using the related calculation $68 + 43$ • use knowledge of place value and of doubles of two-digit whole numbers • partition: double and adjust • partition: add or subtract a whole number and adjust, e.g. $4.3 + 2.9 = 4.3 + 3 - 0.1$, $6.5 - 3.8 = 6.5 - 4 + 0.2$

				<p>150 = 290 using 14 + 15 = 29</p> <ul style="list-style-type: none">• partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)		<ul style="list-style-type: none">• partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times, 12-hour and 24-hour clock)
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Written Methods for Multiplication

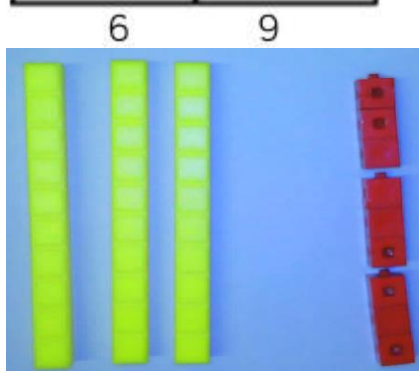
YEAR GROUP & RELEVANT OBJECTIVES	STRATEGY	CONCRETE	PICTORIAL	ABSTRACT / WRITTEN
<p>Y1: Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p> <p>Y2: Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs.</p>	<p>Repeated addition</p>	<p>3×4 $4+4+4$ There are 3 equal groups with 4 in each group.</p> 	<p>Children represent physical resources in a bar model</p> 	<p>$3 \times 4 = 12$ $4+4+4 = 12$</p>
<p>Y1: Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and</p>	<p>Repeated addition on a number line</p>	<p>3×4</p> 	<p>Pictorial representation alongside number line</p> 	<p>Abstract showing jumps of 4 $3 \times 4 = 12$</p> 

<p>arrays with the support of the teacher.</p> <p>Y2: Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.</p>				
<p>Y1: Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p> <p>Y2: Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.</p>	<p>Commutative Law</p> <p>Key vocab: columns and rows (it is important to spend time ensuring children know what each one is)</p>	<p>$2 \times 5 = 5 \times 2$</p>  <p>2 lots of 5 5 lots of 2</p>  <p>Counters and Unifix can be used</p>	<p>Children represent arrays pictorially</p> 	<p>Children are able to write a range of calculations based upon an array</p> <p>Eg.</p> <p>$10 = 2 \times 5$</p> <p>$5 \times 2 = 10$</p> <p>$2 + 2 + 2 + 2 + 2 = 10$</p> <p>$10 = 5 + 5$</p>
<p>Y3: write and calculate</p>	<p>Partition to multiply</p>	<p>4×15</p>	<p>Children represent pictorially</p>	<p>Children work out using jottings</p>

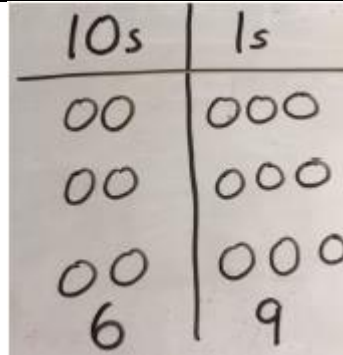
<p>mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods</p> <p>Y4: solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.</p>		 <p>umicon or base10</p>		 <p>Jottings can also be represented on a numberline</p> 
<p>Y4: multiply two-digit and three-digit numbers by a one-digit number using formal written layout</p>	<p>Formal column method</p>	<p>Shown using place value counters 3 x 23</p>	<p>Children represent counters or base10 pictorially</p>	<p>Children record partitioning before moving onto formal method</p> $\begin{array}{r} 3 \times 23 \\ \hline 20 \quad 3 \end{array}$ $\begin{array}{r} 3 \times 20 = 60 \\ 3 \times 3 = 9 \\ \hline 60 + 9 = 69 \end{array}$

Y5: multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Y6: multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication



Base10 can also be used



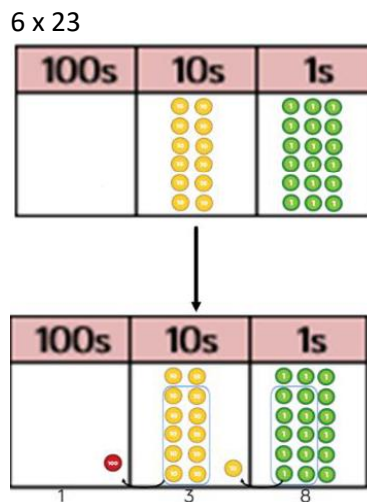
$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Y4: multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Y5: multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

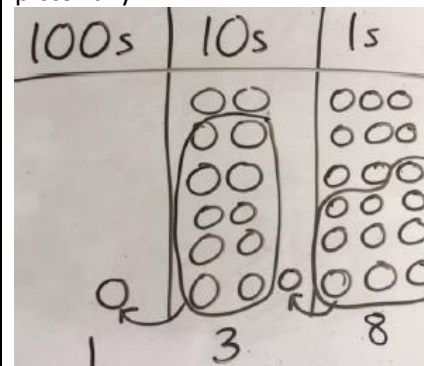
Y6: multiply multi-digit numbers up to

Formal column method
(exchanging across place value columns)



Place value counters

Children represent exchanging pictorially



Formal written method
 $6 \times 23 =$

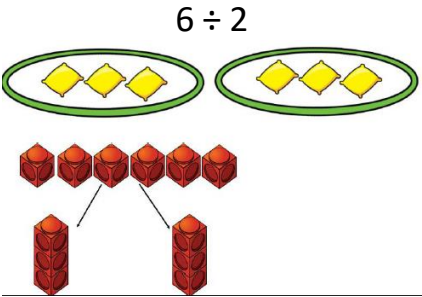
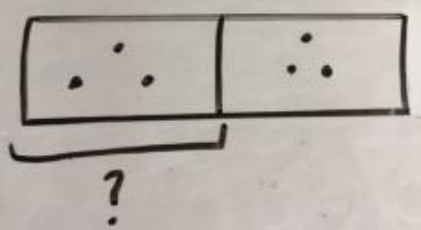
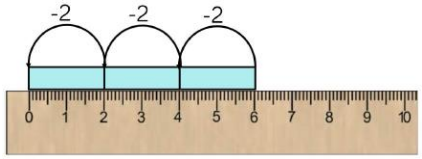
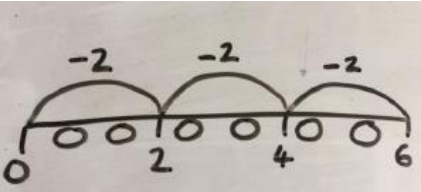
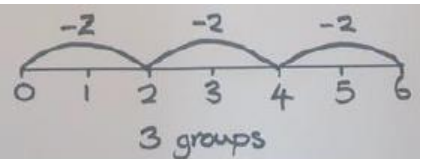
$$\begin{array}{r} 23 \\ \times 6 \\ \hline 138 \\ \hline 11 \end{array}$$



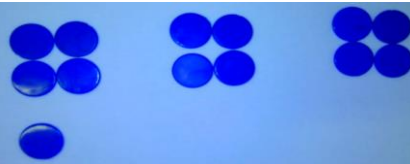
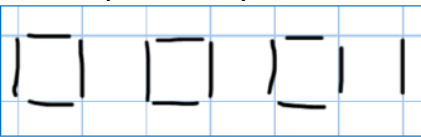
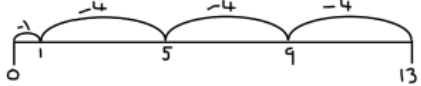
4 digits by a two-digit whole number using the formal written method of long multiplication				
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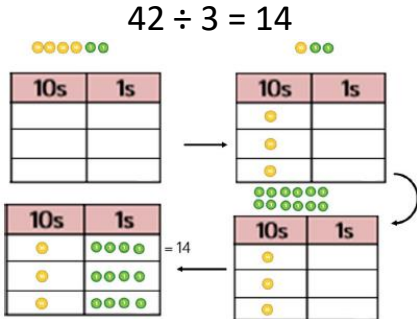
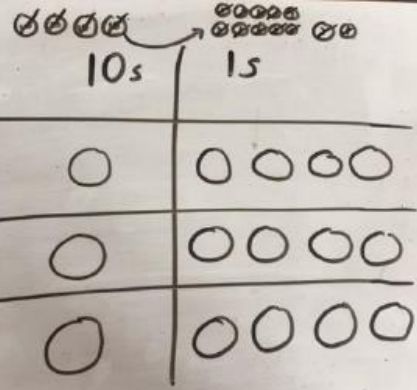
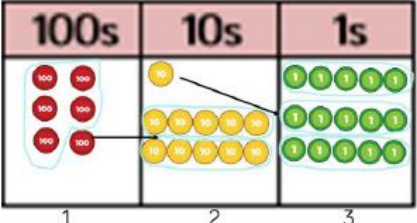
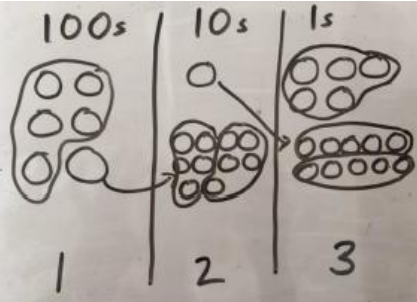
Key vocabulary for multiplication:

double, times, multiplied by, the product of, groups of, lots of, equal groups

Written Methods for Division

YEAR GROUP & RELEVANT OBJECTIVES	STRATEGY	CONCRETE	PICTORIAL	ABSTRACT / WRITTEN		
<p>Y1: Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</p> <p>Y2: Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.</p>	<p>Equal Sharing</p>	 <p>Counters, bean bags, unifix</p>	<p>Children to represent pictorially using the bar model</p> 	$6 \div 2 = 3$ <table border="1" style="margin: auto;"> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">3</td> </tr> </table> <p>Children to be encouraged to make link with 2 times tables</p>	3	3
3	3					
<p>Y1: Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and</p>	<p>Inverse of multiplication (repeated subtraction)</p>	<p>Base10 above a ruler</p> <p>$6 \div 2$</p> 	<p>Children to represent pictorially</p> 	<p>Children represent the equal groups on a number line</p> 		

<p>arrays with the support of the teacher.</p> <p>Y2: Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (\times), division (\div) and equals (=) signs.</p>		 <p>3 groups of 2</p>		
<p>Y3: write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods</p> <p>Y4: use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers</p>	<p>Grouping (with remainders)</p>	<p>Lollipop sticks can be used to make whole shapes (eg dividing by 4 would be squares, 3 would be triangles)</p> <p>$13 \div 4 =$</p>   <p>There are 3 whole squares with 1 left over.</p>	<p>Children represent lollipop sticks pictorially with lines</p>  <p>There are 3 whole squares with 1 left over.</p>	<p>$13 \div 4 = 3$ remainder 1 Encourage children to use their times tables facts.</p> <p>'3 groups of 4 with 1 left over'</p> 

<p>Y5: divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</p>				
<p>Y4: use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers</p>	<p style="text-align: center;">Sharing <i>With place value counters</i></p>	<p style="text-align: center;">$42 \div 3 = 14$</p> 	<p>Children draw place value counters</p> 	<p>Children write process of calculations they have carried out</p> $42 \div 3$ $42 = 30 + 12$ $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$
<p>Y5: divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</p> <p>Y6: divide numbers up to 4 digits by a two-digit whole number using the</p>	<p>Short Division (grouping) <i>Using place value counters to group</i></p>	<p>$615 \div 5$</p>  <ol style="list-style-type: none"> 1. Make 615 with place value counters 2. How many groups of 5 hundreds can you make with 6 hundreds counters? 	<p>Children represent counters pictorially</p> 	<p>Children do calculation using the short division scaffold.</p> $ \begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array} $

<p>formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</p>		<ol style="list-style-type: none"> Exchange 1 hundred for 10 tens How many groups of 5 tens can you make with 11 ten counters? Exchange 1 ten for 10 ones How many groups of 5 ones can you make with 15 ones? 																																									
<p>Y5: divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</p> <p>Y6: divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context</p>	<p>Long division (grouping) <i>Using place value counters</i></p>	<p style="text-align: center;">$2544 \div 12$</p> <p>(children can represent pictorally, however this can become very messy due to the number of exchanges. It is often easier to move straight to the abstract on this occasion)</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">1000s</td> <td style="text-align: center;">100s</td> <td style="text-align: center;">10s</td> <td style="text-align: center;">1s</td> <td></td> </tr> <tr> <td style="text-align: center;">●●</td> <td style="text-align: center;">●●●●● ●●</td> <td style="text-align: center;">●●●●●</td> <td style="text-align: center;">●●●●●</td> <td>We can't group 2 thousands into groups of 12 so will exchange them.</td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">1000s</td> <td style="text-align: center;">100s</td> <td style="text-align: center;">10s</td> <td style="text-align: center;">1s</td> <td></td> </tr> <tr> <td></td> <td style="text-align: center;">●●●●●●●●●● ●●●●●●●●●●</td> <td style="text-align: center;">●●●●●</td> <td style="text-align: center;">●●●●●</td> <td>We can group 24 hundreds into groups of 12 which leaves with 1 hundred.</td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">1000s</td> <td style="text-align: center;">100s</td> <td style="text-align: center;">10s</td> <td style="text-align: center;">1s</td> <td></td> </tr> <tr> <td></td> <td style="text-align: center;">●●●●●●●●●● ●●●●●●●●●●</td> <td style="text-align: center;">●●●●●●●●●●</td> <td style="text-align: center;">●●●●●</td> <td>After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.</td> </tr> </table> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="text-align: center;">1000s</td> <td style="text-align: center;">100s</td> <td style="text-align: center;">10s</td> <td style="text-align: center;">1s</td> <td></td> </tr> <tr> <td></td> <td style="text-align: center;">●●●●●●●●●● ●●●●●●●●●●</td> <td style="text-align: center;">●●●●●●●●●●</td> <td style="text-align: center;">●●●●●●●●●● ●●●●●●●●●●</td> <td>After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.</td> </tr> </table>	1000s	100s	10s	1s		●●	●●●●● ●●	●●●●●	●●●●●	We can't group 2 thousands into groups of 12 so will exchange them.	1000s	100s	10s	1s			●●●●●●●●●● ●●●●●●●●●●	●●●●●	●●●●●	We can group 24 hundreds into groups of 12 which leaves with 1 hundred.	1000s	100s	10s	1s			●●●●●●●●●● ●●●●●●●●●●	●●●●●●●●●●	●●●●●	After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.	1000s	100s	10s	1s			●●●●●●●●●● ●●●●●●●●●●	●●●●●●●●●●	●●●●●●●●●● ●●●●●●●●●●	After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.	<p style="text-align: center;">$2544 \div 12$</p> $\begin{array}{r} 02 \\ 12 \overline{)2544} \\ \underline{24} \\ 1 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ 12 \\ \underline{2} \\ 0212 \\ 12 \overline{)2544} \\ \underline{24} \\ 14 \\ 12 \\ \underline{24} \\ 24 \\ \underline{24} \\ 0 \end{array}$
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Key vocabulary for division:



share, group, divide by, half

Mental Expectations for Multiplication and Division

<u>By the end of the year</u>	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>
<u>-children are able to derive and recall</u>	<ul style="list-style-type: none"> doubles of all numbers to 10, e.g. double 6 odd and even numbers to 20 	<ul style="list-style-type: none"> doubles of all numbers to 20, e.g. double 13, and corresponding halves doubles of multiples of 10 to 50, e.g. double 40, and corresponding halves multiplication facts for the 2, 5 and 10 times-tables, and corresponding division facts odd and even numbers to 100 	<ul style="list-style-type: none"> multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables, and corresponding division facts doubles of multiples of 10 to 100, e.g. double 90, and corresponding halves 	<ul style="list-style-type: none"> multiplication facts to 10×10 and the corresponding division facts doubles of numbers 1 to 100, e.g. double 58, and corresponding halves doubles of multiples of 10 and 100 and corresponding halves fraction and decimal equivalents of one-half, quarters, tenths and hundredths, e.g. 310 is 0.3 and 3100 is 0.03 factor pairs for known multiplication facts 	<ul style="list-style-type: none"> squares to 10×10 division facts corresponding to tables up to 10×10, and the related unit fractions, e.g. $7 \times 9 = 63$ so one-ninth of 63 is 7 and one-seventh of 63 is 9 percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths factor pairs to 100 	<ul style="list-style-type: none"> squares to 12×12 squares of the corresponding multiples of 10 prime numbers less than 100 equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or $35/100$
<u>-working mentally (with jottings where necessary)</u>	<ul style="list-style-type: none"> count on from and back to zero in ones, twos, fives or tens 	<ul style="list-style-type: none"> double any multiple of 5 up to 50, e.g. double 35 halve any multiple of 10 up to 100, e.g. halve 90 find half of even numbers to 40 find the total number of objects when they are organised into groups of 2, 5 or 10 	<ul style="list-style-type: none"> double any multiple of 5 up to 100, e.g. double 35 halve any multiple of 10 up to 200, e.g. halve 170 multiply one-digit or two-digit numbers by 10 or 100, e.g. 7×100, 46×10, 54×100 find unit fractions of numbers and quantities involving halves, thirds, 	<ul style="list-style-type: none"> double any two-digit number, e.g. double 39 double any multiple of 10 or 100, e.g. double 340, double 800, and halve the corresponding multiples of 10 and 100 halve any even number to 200 find unit fractions and simple non-unit fractions of numbers 	<ul style="list-style-type: none"> multiply and divide two-digit numbers by 4 or 8, e.g. 26×4, $96 \div 8$ multiply two-digit numbers by 5 or 20, e.g. 320×5, 14×20 multiply by 25 or 50, e.g. 48×25, 32×50 double three-digit multiples of 10 to 500, e.g. 380×2, and find the corresponding halves, e.g. $760 \div 2$ 	<ul style="list-style-type: none"> multiply pairs of two-digit and single-digit numbers, e.g. 28×3 divide a two-digit number by a single-digit number, e.g. $68 \div 4$ divide by 25 or 50, e.g. $480 \div 25$, $3200 \div 50$ double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2

			quarters, fifths and tenths	<p>and quantities, e.g. 38 of 24</p> <ul style="list-style-type: none"> multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. 325×10, 42×100, $120 \div 10$, $600 \div 100$, $850 \div 10$ multiply a multiple of 10 to 100 by a single-digit number, e.g. 40×3 multiply numbers to 20 by a single-digit, e.g. 17×3 identify the remainder when dividing by 2, 5 or 10 give the factor pair associated with a multiplication fact, e.g. identify that if $2 \times 3 = 6$ then 6 has the factor pair 2 and 3 	<ul style="list-style-type: none"> find the remainder after dividing a two-digit number by a single-digit number, e.g. $27 \div 4 = 6 \text{ R } 3$ multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. 4.3×10, 0.75×100, $25 \div 10$, $673 \div 100$, $74 \div 100$ multiply pairs of multiples of 10, e.g. 60×30, and a multiple of 100 by a single digit number, e.g. 900×8 divide a multiple of 10 by a single-digit number (whole number answers) e.g. $80 \div 4$, $270 \div 3$ find fractions of whole numbers or quantities, e.g. 23 of 27, 45 of 70 kg find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80 find factor pairs for numbers to 100, e.g. 30 has the factor pairs 1×30, 2×15, 3×10 and 5×6 	<ul style="list-style-type: none"> multiply pairs of multiples of 10 and 100, e.g. 50×30, 600×20 divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. $600 \div 20$, $800 \div 400$, $2100 \div 300$ multiply and divide two-digit decimals such as 0.8×7, $4.8 \div 6$ find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g simplify fractions by cancelling scale up and down using known facts, e.g. given that three oranges cost 24p, find the cost of four oranges identify numbers with odd and even numbers of factors and no factor pairs other than 1 and themselves
<u>-understand when to be able to apply</u>	<ul style="list-style-type: none"> use patterns of last digits, e.g. 0 and 5 when counting in fives 	<ul style="list-style-type: none"> partition: double the tens and ones separately, then recombine 	<ul style="list-style-type: none"> partition: when doubling, double the tens and ones 	<ul style="list-style-type: none"> partition: double or halve the tens and ones separately, then recombine 	<ul style="list-style-type: none"> multiply or divide by 4 or 8 by repeated doubling or halving 	<ul style="list-style-type: none"> partition: use partitioning and the distributive law to

		<ul style="list-style-type: none"> • use knowledge that halving is the inverse of doubling and that doubling is equivalent to multiplying by two • use knowledge of multiplication facts from the 2, 5 and 10 times-tables, e.g. recognise that there are 15 objects altogether because there are three groups of five 	<p>separately, then recombine</p> <ul style="list-style-type: none"> • partition: when halving, halve the tens and ones separately, then recombine • use knowledge that halving and doubling are inverse operations • recognise that finding a unit fraction is equivalent to dividing by the denominator and use knowledge of division facts • recognise that when multiplying by 10 or 100 the digits move one or two places to the left and zero is used as a place holder 	<ul style="list-style-type: none"> • use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right and zero is used as a place holder • use knowledge of multiplication facts and place value, e.g. $7 \times 8 = 56$ to find 70×8, 7×80 • use partitioning and the distributive law to multiply, e.g. <ul style="list-style-type: none"> ➤ $13 \times 4 =$ ➤ $(10 + 3) \times 4 =$ ➤ $(10 \times 4) + (3 \times 4) =$ ➤ $40 + 12 = 52$ 	<ul style="list-style-type: none"> • form an equivalent calculation, e.g. to multiply by 5, multiply by 10, then halve; to multiply by 20, double, then multiply by 10 • use knowledge of doubles/halves and understanding of place value, e.g. when multiplying by 50 multiply by 100 and divide by 2 • use knowledge of division facts, e.g. when carrying out a division to find a remainder • use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right relative to the decimal point, and zero is used as a place holder • use knowledge of multiplication and division facts and understanding of place value, e.g. when calculating with multiples of 10 • use knowledge of equivalence between fractions and 	<p>divide tens and ones separately, e.g.</p> <ul style="list-style-type: none"> ➤ $92 \div 4 =$ ➤ $(80 + 12) \div 4 =$ ➤ $20 + 3 = 23$ • form an equivalent calculation, e.g. to divide by 25, divide by 100, then multiply by 4; to divide by 50, divide by 100, then double • use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division • recognise how to scale up or down using multiplication and division, e.g. <ul style="list-style-type: none"> ➤ if three oranges cost 24p: ➤ one orange costs $24 \div 3 = 8p$ ➤ four oranges cost $8 \times 4 = 32p$ • Use knowledge of multiplication and division facts to identify factor pairs and numbers with only two factors
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					<p>percentages, e.g. to find 50%, 25% and 10%</p> <ul style="list-style-type: none">• use knowledge of multiplication and division facts to find factor pairs	
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